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LIGHT CONE RELATIONS FOR THE STRANGENESS CHANGING STRUCTURE FUNCTIONS

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ABSTRACT

Light cone relations are obtained in the scaling limit among the $\Delta S=0$ and $\Delta S=1$ structure functions. The main conclusions are: (i) There are kinematic regions where the $\Delta S=1$ transitions are 7% of the $\Delta S=0$ transitions; (ii) there are differences of the inclusive cross sections which isolate the $\Delta S=1$ transitions and; (iii) application of the results to isoscalar nuclei eliminates the ambiguities arising in the extraction of the neutron data.

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The quark-parton model[1] or its equivalent formulation in terms of the Fritzsch, Gell-Mann[2] conjecture concerning the leading light-cone singularities lead to equalities and inequalities among the structure functions for inelastic electron-proton and neutrino-proton scattering. Such relations have been studied extensively[3,4,5] for $\Delta S = 0$ transitions and will be tested in the forthcoming set of experiments. We discuss here the analogous relations for strangeness changing transitions and point out that they can be useful for estimating the expected event rates. In fact, there are kinematic regions where the $\Delta S = 1$ transitions are as big as 7% of the $\Delta S = 0$ contribution. The results are either in the form of equalities, which become especially useful in certain kinematic regions, or in the form of inequalities valid throughout the deep inelastic region.

Since the two theoretical approaches lead to identical results[4] we can use either of them. We have chosen, for no compelling reason, to present them in the light cone approach. We consider the scattering of a nonet of currents a, b, ..., from an octet of hadrons α, β, ..., belonging to some strong interaction symmetry SU(2) or SU(3):

$$b + \beta \rightarrow a + \alpha$$

The absorptive part of forward scattering is characterized by a familiar set of structure functions. We restrict the discussion to the Bjorken limit[6] $x = q^2/2M\nu$ fixed, $q^2 \rightarrow \infty$; and we denote the structure functions by $(F_i)^{ab}_{\alpha\beta}$. The SU(3) structure of the function is summarized by:

$$(F\pm)^{ab}_{\alpha\beta} = (F_1)^{ab}_{\alpha\beta} \mp \tfrac{1}{2}(F_3)^{ab}_{\alpha\beta}$$
$$= (\pm i f^{abc} + d^{abc})\, G^c_{\pm\alpha\beta} \,. \qquad (1)$$

where the hadron matrix elements $G^c_{\pm}(x)_{\alpha\beta}$ satisfy scaling. The SU(3) indices extend from a = 0, 1, 2, . . . , 8; and the structure constants are given by

$$f^{abo} = 0, \; d^{abo} = \sqrt{2/3} \quad a, b, = 0, 1, \ldots, 8, \qquad (2)$$

while for $c \neq 0$ they are the usual ones found in the literature. In the subsequent discussion we are not concerned with the structure function $F_2(x)$, since it is determined through the Callan-Gross[7] relation:

$$2\, x\, F_1(x) = F_2(x) \qquad (3)$$

The six reduced matrix elements (in a bra, ket notation) that occur in equation (1) are chosen to be

$$< p\,|\,G^3_{\pm}\,|\,p > \; = -< n\,|\,G^3_{\pm}\,|\,n > \qquad (4a)$$
$$< p\,|\,G^0_{\pm}\,|\,p > \; = \; < n\,|\,G^0_{\pm}\,|\,n > \qquad (4b)$$
$$< p\,|\,G^8_{\pm}\,|\,p > \; = \; < n\,|\,G^8_{\pm}\,|\,n > . \qquad (4c)$$

We first consider neutrino and antineutrino induced processes on neutrons and protons with $\Delta S = 0$, denoted by F_i, and $\Delta S = 1$, denoted by S_i. There are sixteen independent quantities expressed in terms of six reduced matrix elements. Of the ten relations that follow four are trivial consequences of charge symmetry. They are:

$$F^{\nu p}_{1,3} = F^{\bar{\nu} n}_{1,3} \tag{5a}$$

$$F^{\nu n}_{1,3} = F^{\bar{\nu} p}_{1,3} \tag{5b}$$

The remaining relations are given by the equations:

$$F^{\nu n}_1 - \tfrac{1}{2}F^{\nu n}_3 - \left(F^{\nu p}_1 - \tfrac{1}{2}F^{\nu p}_3\right) = S^{\bar{\nu} n}_3 - S^{\bar{\nu} p}_3 = 2\left(S^{\bar{\nu} p}_1 - S^{\bar{\nu} n}_1\right) \tag{6a}$$

$$F^{\nu p}_1 + \tfrac{1}{2}F^{\nu p}_3 - \left(F^{\nu n}_1 + \tfrac{1}{2}F^{\nu n}_3\right) = S^{\nu p}_3 - S^{\nu n}_3 = 2\left(S^{\nu p}_1 - S^{\nu n}_1\right) \tag{6b}$$

$$F^{\nu d}_1 + \tfrac{1}{2}F^{\nu d}_3 = S^{\nu d}_1 + \tfrac{1}{2}S^{\nu d}_3 \tag{6c}$$

$$F^{\bar{\nu} d}_1 - \tfrac{1}{2}F^{\bar{\nu} d}_3 = S^{\bar{\nu} d}_1 - \tfrac{1}{2}S^{\bar{\nu} d}_3 \tag{6d}$$

where the superscript d denotes averaging over neutrons and protons. The combinations occuring in equations (6c) and (6d) correspond to the cross-sections for right-handed and left-handed currents and may be easily separable just by themselves. We can also consider the electroproduction structure functions on protons and neutrons, which provide two additional experimental quantities. Since they are also expressed in terms of the same reduced matrix elements, we should obtain two more relations. One deals with a difference between protons and neutrons

$$12\left(F_1^{(\gamma p)} - F_1^{(\gamma n)}\right) = F_3^{(\nu p)} - F_3^{(\nu n)} \tag{7a}$$

This was first obtained by Llewellyn-Smith.[3] The other is a new relation between $\Delta S=1$ and $\Delta S=0$ quantities and involves an averaging over protons and neutrons

$$S_3^{\bar{\nu}d} - S_3^{\nu d} = 12\left(3F_1^{\gamma d} - F_1^{\nu d}\right) \tag{7b}$$

The relations for strangeness-changing contributions can be useful only if there is some experimental technique for separating out the $\Delta S=1$ contribution. In addition to the direct measurement of the strangeness of final states, certain linear combinations of $\Delta S=1$ structure function can be measured directly in inclusive experiments; namely those combinations where the $\Delta S=0$ contribution cancels by virtue of eqs. (5). From Cabibbo theory, the inclusive structure function is expressed in terms of the $\Delta S=0$ and $\Delta S=1$ components as

$$F_{\text{incl.}} = F\cos^2\theta + S\sin^2\theta \tag{8}$$

where F is any structure function and θ is the Cabibbo angle. From eqs. (4) and (8) we obtain

$$S_{1,3}^{\nu p} - S_{1,3}^{\bar{\nu}n} = \frac{1}{\sin^2\theta}\left[F_{1,3}^{\nu p} - F_{1,3}^{\bar{\nu}n}\right]_{\text{incl.}} \tag{9a}$$

$$S_{1,3}^{\nu n} - S_{1,3}^{\bar{\nu}p} = \frac{1}{\sin^2\theta}\left[F_{1,3}^{\nu n} - F_{1,3}^{\bar{\nu}p}\right]_{\text{incl.}} \tag{9b}$$

In practice experiments are performed on protons, deuterons and heavy nuclei and the structure functions for free neutrons are obtained by calculations including Glauber corrections. Some of the errors inherent in this procedure can be avoided by using the deuteron data directly. Relations for deuteron structure functions are obtained by defining a new set of reduced matrix elements analogous to (4) using deuteron states instead of nucleons. There are now only four independent functions instead of six, since the matrix elements of the isovectors $G^3_\pm$ vanish for the isoscalar deuteron. The relations between deuteron structure functions obtained in this way turn out to be exactly equivalent to those obtained for the sums $F^p_{1,3} + F^n_{1,3}$, namely Eqs. (6c), (6d) and (7b) and the sum of Eqs. (5a) and (5b). This is not surprising, since the expressions for the structure functions in terms of reduced matrix elements depend only upon the SU(3) and isospin structure of the states and not on the detailed dynamical structure. The difference between the deuteron and a free neutron and proton appears in the explicit values of the reduced matrix elements. These are of no consequence to this treatment, since we obtain relations between the observed structure functions by eliminating the unknown reduced matrix elements.

Equations (6c), (6d) and (7b) are especially useful in this respect. Substituting Eqs. (9) into Eq. (7a), we obtain:

$$\left[F_3^{\bar{\nu}d} - F_3^{\nu d}\right]_{\text{incl.}} = 12\left(3F_1^{\gamma d} - F_1^{\nu d}\right)\sin^2\theta \tag{10a}$$

Considering Eqs. (6c) and (6d) in the diffractive region, where conventional Regge theory implies the vanishing of F_3, we obtain:

$$F_1^{\bar{\nu}d} = F_1^{\nu d} = S_1^{\nu d} = S_1^{\bar{\nu}d} \tag{10b}$$

This implies that in the Regge Corner of deep inelastic scattering the $\Delta S=1$ contribution accounts for ~ 7% of all the events. In particular for $E \to \infty$

$$\frac{\left.\frac{d\sigma}{dQ^2}\right/ \Delta S=1}{\left.\frac{d\sigma}{dQ^2}\right/ \Delta S=0} = \tan^2\theta \approx 7\% \tag{11}$$

Similarly equation (7b) provides an estimate for the $\Delta S=1$ transitions, as soon as the neutrino structure functions become available.

All other results are in the form of inequatilies. Such results have been discussed in the literature for the case of $\Delta S=0$ transitions. We restrict our discussion to the presentation of the results following from SU(3) symmetry for $\Delta S=1$ transitions and then discussing some of the inequalities. We use the notation of eqs. (21) of Ref. 5 in which the structure functions F_1 and F_3 for $\Delta S=0$ transitions are written in terms of positive definite quantities $U_{\pm}$, $V_{\pm}$ and $W_{\pm}$. We write the structure functions S_1 and S_3 for $\Delta S=1$ transitions in terms of these same parameters.

$$S_1^{\nu p} = 2(U_+ + U_-) \tag{12a}$$

$$S_1^{\nu n} = 2U_+ + U_- + V_- \tag{12b}$$

$$S_1^{\bar{\nu}p} = \tfrac{1}{4}(3U_+ + 2V_+ + 3W_+ + 3U_- + 2V_- + 3W_-) \tag{12c}$$

$$S_1^{\bar{\nu}n} = \tfrac{1}{4}(4U_+ + 4V_+ + 3U_- + 2V_- + 3W_-) \tag{12d}$$

$$S_3^{\nu p} = 4(-U_+ + U_-) \tag{12e}$$

$$S_3^{\nu n} = -4U_+ + 2U_- + 2V_- \tag{12f}$$

$$S_3^{\bar{\nu}p} = \tfrac{1}{2}(-3U_+ - 2V_+ - 3W_+ + 3U_- + 2V_- + 3W_-) \tag{12g}$$

$$S_3^{\bar{\nu}n} = \tfrac{1}{2}(-4U_+ - 4V_+ + 3U_- + 2V_- + 3W_-) \tag{12h}$$

These eqs. (12) can be combined with equations (21) of Ref. 5 to give relations between $\Delta S=1$ and $\Delta S=0$ structure functions. Inequalities follow from the fact that $U_{\pm}$, $V_{\pm}$ and $W_{\pm}$ are positive semi-definite. A large variety of such inequalities are readily constructed from the two sets of equations. One which seems to be particularly useful is

$$\frac{S_1^{\bar{\nu}d}}{F_1^{\nu d}} \geq \frac{1}{2} \tag{13}$$

leading to a lower bound for the strangeness changing contributions of $\sim 3.5\%$.

We have indicated that one can obtain information about the strangeness changing structure functions by looking at inclusive neutrino-nucleon reactions; namely, we can take differences of the inclusive cross-sections that isolate the $\Delta S=1$ contributions by virtue of charge symmetry. The $\Delta S=1$ contribution in some regions is 7% of the $\Delta S=0$ contribution and can be, presumably, observed in the forthcoming experiments. By applying the same analysis to isoscalar nuclei, we obtain inequalities in which the ambiguities arising from the extraction of the neutron data have been eliminated completely. Should any of the inequalities be violated by experiment then it would provide unambiguous evidence for the modification of the Light Cone Algebra and of the Quark-Parton Model.

REFERENCES

[1]R. P. Feynman, Proceedings of Third High Energy Collision Conference at State University of New York, Stony Brook, Gordon and Breach 1970; J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969); P. Landshoff and J. C. Polkinghorne, Nucl. Phys. Rev. B28, 240 (1971); J. Kuti and V. F. Weisskopf, Phys. Rev. D4, 3418 (1971).

[2]H. Fritzsch and M. Gell-Mann, in Broken Scale Invariance and the Light Cone, Edited by M. Gell-Mann, Gordon and Breach, New York, (1971) p. 1. See also Proceedings of the International Conference on Duality and Symmetry in Hadron Physics, Tel Aviv, edited by E. Gotsman, Weizmann Science Press, Jersulam, (1971) p. 317. J. M. Cornwall and R. Jackiw, Phys. Rev. D4, 367 (1971); D. J. Gross and S. B. Treiman, Phys. Rev. D4, 1059 (1971); Y. Frishman, Phys. Rev. Letters, 25, 966 (1970).

[3]C. Llwewllyn-Smith, Nucl. Phys. B17, 277 (1970).

[4]O. Nachtmann, Journal de Physique, 32, 99 (1971); Nuclear Physics, B38, 397 (1972); also Institute for Advanced Study preprint (1971); D. P. Majumdar, Phys. Rev. D3, 2869 (1971).

[5]C. G. Callan, M. Gronau, A. Pais, E. A. Paschos and S. B. Treiman, NAL preprint NAL-THY-29 (1972).

[6]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[7]C. G. Callan and D. J. Gross, Phys. Rev. Lett. 21, 311 (1968).